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NONEQUILIBRIUM QUANTUM SCALAR FIELDS IN COSMOLOGY[1]

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We elaborate further the functional Schrödinger-picture approach to the quantum field in curved spacetimes using the generalized invariant method and construct explicitly the Fock space, which we relate with the thermal field theory. We apply the method to a free massive scalar field in the de Sitter spacetime, and find the exact quantum states, construct the Fock space, and evaluate the two-point function and correlation function.

1 Introduction

Quantum field theory in curved spacetimes (QFTCS) has been a hot issue revisited recently and applied widely to cosmology. There have been two conventional canonical approaches to QFTCS: one is based on the solution space of classical equations [1] and the other is the functional Schrödinger-picture field theory [2]. One may show that these two canonical methods give the identical results in the end. The other nonconventional approach is the semiclassical gravity, a kind of QFTCS, in which not only the quantum back reaction of matter field but also the quantum gravitational corrections to matter field are considered and has been applied successfully and usefully to cosmology, derived from the Wheeler-DeWitt equation (see [3] for a self-consistent method and recent references). But this does not imply that the canonical quantum gravity based on the Wheeler-DeWitt equation is free of all the conceptual and technical problems.

If quantum gravity written formally in the form $\hat{G}_{\mu\nu} = 8\pi\hat{T}_{\mu\nu}$ succeeds indeed, there should be some limiting procedures from quantum gravity to semiclassical gravity written formally in the form $G_{\mu\nu} = 8\pi\left\langle\hat{T}_{\mu\nu}\right\rangle$, in which the background gravity treated as classical is equated with the expectation value of quantum energy-momentum tensor. It should be remarked that regardless of approaches QFTCS is characterized by time-dependence due to the time-

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dependent metrics of spacetimes in general, that is, quantum fields are nonequilibrium or out-of-equilibrium.

In this talk we shall elaborate the functional Schrödinger-picture approach further by unifying the generalized invariant method with the functional Schrödinger-picture approach to quantum field theory in curved spacetimes. One of the motivations for introducing the functional Schrödinger-picture approach was its effectiveness and usefulness in treating these time-dependent Hamiltonian systems of fields. On the other hand, the generalized invariant method has been used to find explicitly the exact quantum states of time-dependent Hamiltonian systems in quantum mechanics [4]. Although these two methods were introduced largely for time-dependent quantum field theory and quantum mechanical systems, there has not been yet any attempt to combine these two approaches in particular to a field theoretical framework. The method introduced in this talk applies equally to not only the conventional canonical approach in functional Schrödinger-picture but also the semiclassical gravity from the Wheeler-DeWitt equation.

2 Functional Schrödinger Equation

To simplify the model under study, we shall consider a free massive scalar field in a spatially flat ($k = 1$) Friedmann-Robertson-Walker universe with the metric

$$ds^2 = -dt^2 + R^2(t)d\mathrm{x}^2. \tag{1}$$

The Hamiltonian for the free massive scalar field takes the form

$$H(t) = \int d\mathrm{x} \left[\frac{1}{2}\frac{1}{R^3(t)}\pi^2(\mathrm{x}, t) + \frac{1}{2}R(t)(\nabla\phi(\mathrm{x}, t))^2 + \frac{1}{2}R^3(t)\left(m^2 + \xi\mathbf{R}\right)\phi^2(\mathrm{x}, t)\right], \tag{2}$$

where $\mathbf{R}$ is the three-curvature. $\xi = 0$ is the minimal scalar field and $\xi = 1/6$ is the conformal scalar field. We decompose the scalar field into modes by confining it into a box of size l and use the orthonormal basis

$$u^{(+)}_{\mathbf{n}}(\mathbf{x}) = \sqrt{2/l^3}\cos\left(2\pi\mathbf{n}\cdot\mathbf{x}/l\right), u^{(-)}_{\mathbf{n}}(\mathbf{x}) = \sqrt{2/l^3}\sin\left(2\pi\mathbf{n}\cdot\mathbf{x}/l\right). \tag{3}$$

In a compact notation, $\sum_{\pm,\mathbf{n}} = \sum_\alpha$, the scalar field and its momentum can be expanded as

$$\phi(\mathbf{x}, t) = \sum_\alpha \phi_\alpha(t)u_\alpha(\mathbf{x}), \pi(\mathbf{x}, t) = \sum_\alpha \pi_\alpha(t)u_\alpha(\mathbf{x}). \tag{4}$$

Then one obtains the mode-decomposed Hamiltonian

$$H(t) = \sum_{\alpha} H_{\alpha}(t) = \sum_{\alpha} \left[\frac{1}{2} \frac{1}{R^3(t)} \pi_{\alpha}^2 + \frac{1}{2} R^3(t) \left(m^2 + \xi \mathbf{R} + \frac{\mathbf{k}^2}{R^2(t)} \right) \phi_{\alpha}^2 \right]. \tag{5}$$

The free massive scalar field leads to the Hamiltonian which is a collection of the time-dependent harmonic oscillators with the time-dependent mass and frequency squared

$$M(t) = R^3(t), \omega_{\alpha}^2(t) = m^2 + \xi \mathbf{R} + \mathbf{k}^2 / R^2(t). \tag{6}$$

We quantize the scalar field according to the Schrödinger-picture:

$$i\hbar \frac{\partial}{\partial t} \Psi(\phi_{\alpha}, t) = \hat{H}(t) \Psi(\phi_{\alpha}, t). \tag{7}$$

The Hamiltonian being mode-decomposed, the total wave function for the scalar field is

$$\Psi(\phi_{\alpha}, t) = \Pi_{\alpha} \Psi_{n_{\alpha}}(\phi_{\alpha}, t) \tag{8}$$

a product of the wave function $\Psi_{n_{\alpha}}(\phi_{\alpha}, t)$ for each oscillator $H_{\alpha}(t)$.

3 Fock Space out of Generalized Invariant

In order to find the exact quantum states and construct the Fock space for the Hamiltonian (5), we shall use the generalized invariant method and advance it to a field theoretical framework. The most advantageous point of the generalized invariant is the substantiation and easiness in constructing the Fock space of exact quantum states and readiness in applying to the density operator. From the well-known result of the generalized invariant called Lewis-Riesenfeld invariant [4], the exact quantum state of the α-th oscillator's Schrödinger equation (7) is given by

$$\Psi_{n_{\alpha}}(\phi_{\alpha}, t) = \exp \left(- \int \langle \alpha, n_{\alpha}, t | \frac{\partial}{\partial t} - \frac{1}{i\hbar} \hat{H}_{\alpha}(t) | \alpha, n_{\alpha}, t \rangle \, dt \right) | \alpha, n_{\alpha}, t \rangle \tag{9}$$

in terms of the eigenstates of the generalized invariant

$$\hat{I}_{\alpha}(t) | \alpha, n_{\alpha}, t \rangle = \lambda_{\alpha n_{\alpha}} | \alpha, n_{\alpha}, t \rangle , \tag{10}$$

with a time-independent eigenvalue $\lambda_{\alpha n_{\alpha}}$, which satisfies the invariant equation

$$\frac{\partial}{\partial t} \hat{I}_{\alpha}(t) - \frac{i}{\hbar} \left[\hat{I}_{\alpha}(t), \hat{H}_{\alpha}(t) \right] = 0. \tag{11}$$

The most simplest way to construct the Fock space is to find the set of the first order fundamental invariants of the form

$$\hat{I}_{\alpha j} = \phi_{\alpha j}(t)\hat{\pi}_\alpha - R^3(t)\dot{\phi}_{\alpha j}(t)\hat{\phi}_\alpha \tag{12}$$

where j runs for 1,2 and

$$\frac{\partial^2}{\partial t^2}\phi_{\alpha j}(t) + 3\frac{\dot{R}(t)}{R(t)}\frac{\partial}{\partial t}\phi_{\alpha j}(t) + \left(m^2 + \xi\mathbf{R} + \frac{\mathbf{k}^2}{R^2(t)}\right)\phi_{\alpha j}(t) = 0, \tag{13}$$

are classical solutions of the α-th oscillator. We can make the first order invariants satisfy the commutation relation

$$[\hat{I}_{\alpha 2}(t), \hat{I}_{\beta 1}(t)] = \hbar\delta_{\alpha\beta} \tag{14}$$

by imposing a condition on the integration constants such that

$$R^3(t)\left(\phi_{\alpha 1}(t)\dot{\phi}_{\alpha 2}(t) - \phi_{\alpha 2}(t)\dot{\phi}_{\alpha 1}(t)\right) = i. \tag{15}$$

We see the group structure $SU^\infty(1,1)$ for the scalar field by forming the second order independent invariants

$$\hat{I}^{(2)}_{\alpha jk}(t) = \frac{1}{2}\left(\hat{I}_{\alpha j}(t)\hat{I}_{\alpha k}(t) + \hat{I}_{\alpha k}(t)\hat{I}_{\alpha j}(t)\right). \tag{16}$$

and by redefining the basis

$$\hat{K}_{\alpha +}(t) = \frac{1}{2\hbar}\hat{I}^{(2)}_{\alpha 11}(t), \hat{K}_{\alpha -}(t) = \frac{1}{2\hbar}\hat{I}^{(2)}_{\alpha 22}(t), \hat{K}_{\alpha 0}(t) = \frac{1}{2\hbar}\hat{I}^{(2)}_{\alpha 12}(t). \tag{17}$$

such that

$$\left[\hat{K}_{\alpha 0}(t), \hat{K}_{\beta\pm}(t)\right] = \pm\hat{K}_{\pm}(t)\delta_{\alpha\beta}, \left[\hat{K}_{\alpha +}(t), \hat{K}_{\beta -}(t)\right] = -2\hat{K}_{\alpha 0}(t)\delta_{\alpha\beta}. \tag{18}$$

The strong point of the commutation relation (14) is the possibility to interpret the first order invariants $\hat{I}_{\alpha 1}(t)$ and $\hat{I}_{\alpha 2}(t)$ as a kind of formal creation and annihilation operators for a time-dependent oscillator

$$\hat{A}^\dagger_\alpha(t) = \hat{I}_{\alpha 1}(t), \hat{A}_\alpha(t) = \hat{I}_{\alpha 2}(t), \tag{19}$$

by imposing further a condition on complex classical solutions

$$\mathrm{Im}\left(\frac{\dot{\phi}_{\alpha 1}(t)}{\phi_{\alpha 1}(t)}\right) < 0, \tag{20}$$

In an asymptotic region in which the mass and frequency squared tend to constant values, the formal creation and annihilation operators corresponds to the conventional creation and

annihilation operators up to some trivial time-dependent phase factors. From the group structure the operator $\hat{K}_{\alpha+}$ ($\hat{K}_{\alpha-}$) raises (lowers) the eigenstates of $\hat{K}_{\alpha 0}$ by two photons.

The ground state wave function that should be annihilated by $\hat{A}_\alpha(t)$ is given by

$$\Psi_{\alpha 0}(\phi_\alpha, t) = \left(\frac{1}{2\pi\hbar|\phi_{\alpha 1}(t)|^2}\right)^{\frac{1}{4}} \exp\left(\frac{iR^3(t)\dot{\phi}^*_{\alpha 1}(t)}{2\hbar\phi^*_{\alpha 1}(t)}\phi_\alpha^2\right). \tag{21}$$

The n_αth wave function can be obtained by applying the operator $\hat{A}^{n_\alpha \dagger}(t)$ to the ground state wave function

$$\Psi_{n_\alpha}(\phi_\alpha, t) = \left(\frac{1}{n_\alpha!(\sqrt{\hbar})^{n_\alpha}}\right)^{\frac{1}{2}} \hat{A}_\alpha^{\dagger n_\alpha}(t)\Psi_{\alpha 0}(\phi_\alpha, t). \tag{22}$$

Now one finds relatively easily the dispersion relations for each oscillator

$$\begin{aligned}(\Delta\phi_\alpha)^2_{n_\alpha} &= \phi_{\alpha 1}(t)\phi^*_{\alpha 1}(t)\hbar(2n_\alpha + 1), &(23)\\ (\Delta\pi_\alpha)^2_{n_\alpha} &= R^3(t)\dot{\phi}_{\alpha 1}(t)\dot{\phi}^*_{\alpha 1}(t)\hbar(2n_\alpha + 1), &(24)\end{aligned}$$

and the expectation value of the Hamiltonian operator

$$\left\langle \hat{H}_\alpha(t)\right\rangle_{n_\alpha} = \frac{R^3(t)}{2}\left(\dot{\phi}_{\alpha 1}(t)\dot{\phi}^*_{\alpha 1}(t) + \left(m^2 + \xi\mathbf{R} + \frac{\mathbf{k}^2}{R^2(t)}\right)\phi_{\alpha 1}(t)\phi^*_{\alpha 1}(t)\right)\hbar(2n_\alpha + 1). \tag{25}$$

One particular feature of the quantum states found in this paper is that the expectation value of the Hamiltonian, the time-time component of the energy-momentum tensor, is proportional to that of the classical one [5].

One also finds the Bogoliubov transformation between the formal creation and annihilation operators at two different times

$$\begin{aligned}\hat{A}^\dagger_\alpha(t) &= u_{\alpha 1}\hat{A}^\dagger_\alpha(t_0) + u_{\alpha 2}\hat{A}_\alpha(t_0),\\ \hat{A}_\alpha(t) &= u^*_{\alpha 2}\hat{A}^\dagger_\alpha(t_0) + u^*_{\alpha 1}\hat{A}_\alpha(t_0),\end{aligned} \tag{26}$$

where

$$\begin{aligned}u_{\alpha 1}(t, t_0) &= iR^3(t)\left(\dot{\phi}_{\alpha 1}(t)\phi^*_{\alpha 1}(t_0) - \phi_{\alpha 1}(t)\dot{\phi}^*_1(t_0)\right),\\ u_{\alpha 2}(t, t_0) &= iR^3(t)\left(\phi_{\alpha 1}(t)\dot{\phi}_{\alpha 1}(t_0) - \dot{\phi}_{\alpha 1}(t)\phi_{\alpha 1}(t_0)\right).\end{aligned} \tag{27}$$

By direct substitution one can show that

$$|u_{\alpha 1}(t, t_0)|^2 - |u_{\alpha 2}(t, t_0)|^2 = 1. \tag{28}$$

Then there is a unitary transformation of the formal creation and annihilation operators at two different times

$$\hat{A}^{\dagger}_{\alpha}(t) = \hat{S}^{\dagger}_{\alpha}(t,t_0)\hat{A}^{\dagger}_{\alpha}(t_0)\hat{S}_{\alpha}(t,t_0) \tag{29}$$

where

$$\hat{S}_{\alpha}(t,t_0) = \exp\left(i\theta_{\alpha 1}\hat{A}^{\dagger}_{\alpha}(t_0)\hat{A}_{\alpha}(t_0)\right)\exp\left(\frac{\nu_{\alpha}}{2}e^{-i(\theta_{\alpha 1}-\theta_{\alpha 2})}\hat{A}^{2\dagger}_{\alpha}(t_0) - \text{h.c.}\right) \tag{30}$$

is the squeeze operator, in which

$$u_{\alpha 1}(t,t_0) = \cosh\nu_{\alpha}e^{-i\theta_{\alpha 1}}, u_{\alpha 2}(t,t_0) = \sinh\nu_{\alpha}e^{-i\theta_{\alpha 2}}. \tag{31}$$

We exploit the physical meaning of the time-dependent vacuum state $|0,t\rangle$ that is annihilated by all the formal annihilation operators

$$\hat{A}_{\alpha}(t)\,|0,t\rangle = 0. \tag{32}$$

Thus the vacuum state is the tensor product of each ground state

$$|0,t\rangle = \Pi_{\alpha}\otimes|\alpha,0,t\rangle\,. \tag{33}$$

From the Bogoliubov transformation (26) it follows that the inner product of the ground state of each mode at two different times is given by

$$\langle\alpha,0,t_0|\alpha,0,t\rangle = \left(\frac{1}{|u_{\alpha}(t,t_0)|}\right)^{\frac{1}{2}} \tag{34}$$

and in the continuum limit that of the field at two different times is given by

$$\langle 0,t_0|0,t\rangle = \exp\left(-\frac{1}{2}\frac{l^3}{(2\pi)^3}\int d\mathbf{k}\ln|u_{\alpha 1}(t,t_0)|\right). \tag{35}$$

Noting that $|u_{\alpha 1}(t,t_0)| \leq 1$, in which the equality holds only when $t = t_0$, this implies that in the infinite volume limit the time-dependent vacuum state is orthogonal each other at two different times

$$\langle 0,t_0|0,t\rangle = 0. \tag{36}$$

Thus we obtain explicitly the infinitely many unitarily inequivalent Fock representations[2] [6]. In this sense our functional Schrödinger-picture approach to quantum fields in curved spacetimes substantiates the Fock space used in the thermal field theory.

[2] It was pointed out by F. C. Khanna and G. Vitiello that the new vacuum (33) makes sense in spite of the time-dependence and may have a relation with the one parameter-dependent vacuum in the thermal field theory that leads to the infinitely many unitarily inequivalent Fock space. This point suggests strongly that our functional Schrödinger-picture approach may have a close connection with the thermal field theory.

The instability of the Fock space, that is the unitary inequivalence of the vacuum at any two different times originated from time-dependent metrics leads to the particle creation

$$\langle 0,t|\,\hat{N}(t)\,|0,t\rangle = \sum_{\alpha} \langle \alpha,0,t|\,\hat{N}_{\alpha}(t)\,|\alpha,0,t\rangle = \frac{l^3}{(2\pi)^3}\int d\mathbf{k}|u_2(k,t,t_0)|^2. \tag{37}$$

Thus we recover in the functional Schrödinger-picture approach the cosmological particle creation first introduced by Parker [7].

4 Physical Application

We apply the results in the previous sections to a free massive scalar field in the deSitter spacetime, a spatially flat Friedmann-Robertson-Walker universe with the metric

$$ds^2 = -dt^2 + e^{2H_0t}d\mathbf{x}^2, \tag{38}$$

where H_0 is the expansion rate driven by the vacuum energy density. Then one obtains the Hamiltonian

$$H(t) = \sum_{\alpha}\left[\frac{1}{2}e^{-3H_0t}\pi_{\alpha}^2 + \frac{1}{2}e^{3H_0t}\left(m^2 + 12\xi H_0^2 + \mathbf{k}^2e^{-2H_0t}\right)\phi_{\alpha}^2\right]. \tag{39}$$

Each mode of the scalar field has the mass and frequency squared

$$M(t) = e^{3H_0t}, \omega_{\alpha}^2(t) = m^2 + 12\xi H_0^2 + \mathbf{k}^2e^{-2H_0t}. \tag{40}$$

The classical equation of motion

$$\ddot{\phi}_{\alpha}(t) + 3H_0\dot{\phi}_{\alpha} + \left(m^2 + 12\xi H_0^2 + \mathbf{k}^2e^{-2H_0t}\right)\phi_{\alpha}(t) = 0, \tag{41}$$

has two independent complex solutions

$$\phi_{\alpha_1}(t) = \sqrt{\frac{\pi}{4H_0}}e^{-\frac{3}{2}H_0t}H_{\nu}^{(1)}(z), \phi_{\alpha_2}(t) = \sqrt{\frac{\pi}{4H_0}}e^{-\frac{3}{2}H_0t}H_{\nu}^{(2)}(z), \tag{42}$$

where $H_{\nu}^{(1)}$ and $H_{\nu}^{(2)}$ are the Hankel functions of the first and second kind respectively, and

$$\nu = \left(\frac{9}{4} - \frac{m^2 + 12\xi H_0^2}{H_0^2}\right)^{\frac{1}{2}}, z = \frac{k}{H_0}e^{-H_0t}. \tag{43}$$

Following Sec. 3, we find explicitly the particular invariants in terms of classical solutions by

$$\begin{aligned}\hat{I}_{\alpha 1}(t) &= \phi_{\alpha 1}(t)\hat{\pi}_{\alpha} - e^{3H_0t}\dot{\phi}_{\alpha 1}(t)\hat{\phi}_{\alpha},\\ \hat{I}_{\alpha 2}(t) &= \phi_{\alpha 2}(t)\hat{\pi}_{\alpha} - e^{3H_0}\dot{\phi}_{\alpha 2}(t)\hat{\phi}_{\alpha}.\end{aligned} \tag{44}$$

The first order invariants act as the time-dependent creation and annihilation operators

$$\hat{I}_{\alpha 1}(t) = \hat{A}^{\dagger}_{\alpha}(t), \hat{I}_{\alpha 2}(t) = \hat{A}_{\alpha}(t), \tag{45}$$

such that

$$\left[\hat{A}_{\alpha}(t), \hat{A}^{\dagger}_{\alpha}(t)\right] = \hbar \tag{46}$$

The ground state is annihilated by the annihilation operator

$$\hat{A}_{\alpha}(t) \left|\alpha, 0, t\right\rangle = 0, \tag{47}$$

and the n_αth states are obtained by applying the creation operators n_α times

$$\left|\alpha, n_{\alpha}, t\right\rangle = \frac{1}{\sqrt{n_{\alpha}!}} \left(\hat{A}^{\dagger}_{\alpha}(t)\right)^{n_{\alpha}} \left|\alpha, 0, t\right\rangle . \tag{48}$$

It is worthy to note that the new vacuum state (33) constructed here for the de Sitter spacetime is nothing but the Bunch-Davies vacuum state[3] [9].

From the field operators at an equal time

$$\hat{\phi}(\mathbf{x}, t) = \sum_{\alpha} \hat{\phi}_{\alpha}(t) u_{\alpha}(\mathbf{x}), \hat{\phi}(\mathbf{y}, t) = \sum_{\beta} \hat{\phi}_{\beta}(t) u_{\beta}(\mathbf{x}), \tag{49}$$

we find the two-point function evaluated with respect to the ground state

$$\left\langle \hat{\phi}(\mathbf{x}, t)\hat{\phi}(\mathbf{y}, t)\right\rangle = \sum_{\alpha} u_{\alpha}(\mathbf{x}) u_{\alpha}(\mathbf{y}) \phi_{\alpha 2}(t) \phi^{*}_{\alpha 2}(t). \tag{50}$$

In the infinite volume limit, it takes the form

$$\left\langle \hat{\phi}(\mathbf{x}, t)\hat{\phi}(\mathbf{y}, t)\right\rangle = \frac{1}{(2\pi)^3} \int d\mathbf{k} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} |\phi(\mathbf{k}, t)|^2, \tag{51}$$

where

$$\phi(\mathbf{k}, t) = \phi_{\alpha 2}(t) = \sqrt{\frac{\pi}{4H_0}} e^{-\frac{3}{2}H_0 t} H^{(1)}_{\nu}(z). \tag{52}$$

One reads the correlation function from the two-point function

$$\begin{aligned} \left\langle \hat{\phi}^2(\mathbf{x}, t)\right\rangle &= \frac{1}{(2\pi)^3} \int d\mathbf{k} |\phi(\mathbf{k}, t)|^2 \\ &= \frac{H_0}{8\pi} e^{-H_0 t} \int_0^{\infty} dz z \left(H_0^2 e^{2H_0 t} z^2 - \gamma^2\right)^{\frac{1}{2}} |H^{(1)}_{\nu}(z)|^2, \end{aligned} \tag{53}$$

which goes to infinity due to long wavelength modes and can be regularized using the covariant point-splitting method to give proper physical quantity [9].

[3] This supports our earlier argument [8] that the ground state (33) of scalar field may play the role of the physical vacuum extending the Minkowski, Bunch-Davies, and Hawking vacua.

One also finds explicitly the dispersion relations between the n_αth quantum state

$$\begin{aligned}(\Delta\hat{\pi}_\alpha)^2_{n_\alpha} &= \frac{\pi H_0}{4} e^{3H_0 t}\left(\frac{3}{2}+z\right)^2 \left|zH^{(1)}_{\nu+1}(z)+\nu H^{(1)}_\nu(z)\right|^2 \hbar(2n_\alpha+1),\\ (\Delta\hat{\phi}_\alpha)^2_n &= \frac{\pi}{4H_0} e^{-3H_0 t}|H^{(1)}_\nu(z)|^2\hbar(2n_\alpha+1),\end{aligned} \tag{54}$$

and the uncertainty relation

$$(\Delta\hat{\pi}_\alpha)_{n_\alpha}(\Delta\hat{\phi}_\alpha)_{n_\alpha} = \frac{\pi}{4}\left(\frac{3}{2}+z\right)\left|H^{(1)}_\nu(z)\right|\left|zH^{(1)}_{\nu+1}(z)+\nu H^{(1)}_\nu(z)\right|\hbar(2n_\alpha+1). \tag{55}$$

At an early stage of expansion, $t \to -\infty$ ($z \to \infty$), the uncertainty relation takes the minimum value $(\Delta\hat{\pi}_\alpha)_{n_\alpha}(\Delta\hat{\phi}_\alpha)_{n_\alpha} \simeq \hbar(2n_\alpha+1)/2$, which means that the adiabatic theorem holds. But at a later stage of expansion, $t \to \infty$ ($z \to 0$), the uncertainty relation increases indefinitely by an exponential factor $e^{2\nu H_0 t}$. The expectation value of the Hamiltonian operator is given explicitly by

$$\left\langle \hat{H}_\alpha(t)\right\rangle_{n_\alpha} = \frac{\pi H_0}{8}\left[\left|(\frac{3}{2}+\nu)H^{(1)}_\nu(z)+zH^{(1)}_{\nu+1}(z)\right|^2+(\frac{9}{4}-\nu^2+z^2)|H^{(1)}_\nu(z)|^2\right]\hbar(2n_\alpha+1). \tag{56}$$

$\left\langle \hat{H}(t)\right\rangle$ diverges due to long wavelenth modes and should be regularized to give the correct back reaction $G_{00} = 8\pi\left\langle \hat{H}(t)\right\rangle$. The number operator for each mode is

$$\hat{N}_\alpha(t) = \hat{A}^\dagger_\alpha(t)\hat{A}_\alpha(t), \tag{57}$$

and the number of particles created in each mode during the evolution of field from an initial time t_0 to a final time t is

$$\begin{aligned}\langle\alpha,0,t|\,\hat{N}_\alpha(t)\,|\alpha,0,t_0\rangle &= \frac{\pi^2}{16}e^{3H_0(t-t_0)}\Big|\Big((\frac{3}{2}+\nu)H^{(1)}_\nu(z)+zH^{(1)}_{\nu+1}(z)\Big)H^{(1)}_\nu(z_0)\\ &\quad -\Big((\frac{3}{2}+\nu)H^{(1)}_\nu(z_0)+z_0H^{(1)}_{\nu+1}(z_0)\Big)H^{(1)}_\nu(z)\Big|^2.\end{aligned} \tag{58}$$

One can show that the Bogoliubov transformation and the number of particles created therefrom is the same as those obtained from the canonical approach based on the classical solution space of the Klein-Gordon equation by taking the correspondence $e^{3H_0t/2}\phi_{\alpha 1}(t) \leftrightarrow f(t)$ and interpreting as $e^{3H_0t/2}\phi_{\alpha 1}(t) \leftrightarrow f^{\rm out}(t), e^{3H_0t/2}\phi_{\alpha 1}(t_0) \leftrightarrow f^{\rm in}(t_0)$. A particularly useful method for the particle creation and entropy production is to introduce the squeeze operator (31). The amount of increase of the information theoretic entropy is given explicitly by

$$\Delta S_{\rm ent} = \sum_\alpha 2|\nu_\alpha|. \tag{59}$$

In summary, we are able to construct explicitly the Fock space of quantum fields in curved spacetimes, whose vacuum state provides the one parameter-dependent unitarily inequivalent Fock representations of the thermal field theory. Futhermore, the two-point and correlation function, the back reaction of the energy-momentum tensor, and the particle creation number are explicitly calculable. The field theoretical method develpoed in this talk can be applied equally to the nonequilibrium quantum field theory in curved spacetimes derived from the Wheeler-DeWitt equation, in which one is able to account self-consistently both the quantum back reaction of matter fields and the quantum gravitational corrections to the field equation.

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